# CBCS SCHEME

USN | | | | | | | | | | |

15MAT31

## Third Semester B.E. Degree Examination, Dec.2019/Jan.2020
## Engineering Mathematics – III

Time: 3 hrs.  Max. Marks: 80

***Note: Answer any FIVE full questions, choosing ONE full question from each module.***



### Module-1

**1** a. Obtain the Fourier expansion of the function f(x) = x over the interval $(-\pi, \pi)$. Deduce that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots..$ **(08 Marks)**

b. The following table gives the variations of a periodic current A over a certain period T:

| t (sec) | 0 | T/6 | T/3 | T/2 | 2T/3 | 5T/6 | T |
|---|---|---|---|---|---|---|---|
| A (amp) | 1.98 | 1.30 | 1.05 | 1.30 | -0.88 | -0.25 | 1.98 |

Show that there is a direct current part of 0.75amp in the variable current and obtain the amplitude of the first harmonic. **(08 Marks)**

**OR**

**2** a. Obtain the Fourier series for the function $f(x) = 2x - x^2$ in $0 \le x \le 2$. **(06 Marks)**

b. Represent the function

$$f(x) = \begin{cases} x, & \text{for } 0 < x < \pi/2 \\ \pi/2 & \text{for } \pi/2 < x < \pi \end{cases}$$

in a half range Fourier sine series. **(05 Marks)**

c. Determine the constant term and the first cosine and sine terms of the Fourier series expansion of y from the following data:

| x° | 0 | 45 | 90 | 135 | 180 | 225 | 270 | 315 |
|---|---|---|---|---|---|---|---|---|
| y | 2 | 3/2 | 1 | 1/2 | 0 | 1/2 | 1 | 3/2 |

**(05 Marks)**

### Module-2

**3** a. Find the complex Fourier transform of the function

$$f(x) = \begin{cases} 1 & \text{for } |x| \le a \\ 0 & \text{for } |x| > a \end{cases} \quad \text{Hence evaluate } \int_0^\infty \frac{\sin x}{x} dx.$$ **(06 Marks)**

b. If $\bar{u}(z) = \frac{2z^2 + 3z + 12}{(z-1)^4}$ show that $u_0 = 0 \quad u_1 = 0 \quad u_2 = 2 \quad u_3 = 11$. **(05 Marks)**

c. Obtain the Fourier cosine transform of the function

$$f(x) = \begin{cases} 4x, & 0 < x < 1 \\ 4 - x, & 1 < x < 4 \\ 0 & x > 4 \end{cases}$$ **(05 Marks)**

**OR**

4 a. Obtain the Z-transform of $\cos n\theta$ and $\sin n\theta$. **(06 Marks)**

b. Find the Fourier sine transform of $f(x) = e^{-|x|}$ and hence evaluate $\int_0^\infty \frac{x \sin mx}{1+x^2} dx \quad m > 0$. **(05 Marks)**

c. Solve by using Z-transforms $y_{n+2} + 2y_{n+1} + y_n = n$ with $y_0 = 0 = y_1$. **(05 Marks)**

## Module-3

5 a. Fit a second degree parabola $y = ax^2 + bx + c$ in the least square sense for the following data and hence estimate y at x = 6. **(06 Marks)**

| | | | | | |
|---|---|---|---|---|---|
| x | 1 | 2 | 3 | 4 | 5 |
| y | 10 | 12 | 13 | 16 | 19 |

b. Obtain the lines of regression and hence find the coefficient of correlation for the data:

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| x | 1 | 3 | 4 | 2 | 5 | 8 | 9 | 10 | 13 | 15 |
| y | 8 | 6 | 10 | 8 | 12 | 16 | 16 | 10 | 32 | 32 |

**(05 Marks)**

c. Use Newton-Raphson method to find a real root of $x\sin x + \cos x = 0$ near $x = \pi$. Carryout the iterations upto four decimal places of accuracy. **(05 Marks)**

**OR**

6 a. Show that a real root of the equation $\tan x + \tanh x = 0$ lies between 2 and 3. Then apply the Regula Falsi method to find third approximation. **(06 Marks)**

b. Compute the coefficient of correlation and the equation of the lines of regression for the data:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| x | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| y | 9 | 8 | 10 | 12 | 11 | 13 | 14 |

**(05 Marks)**

c. Fit a curve of the form $y = ae^{bx}$ for the data:

| | | | |
|---|---|---|---|
| x | 0 | 2 | 4 |
| y | 8.12 | 10 | 31.82 |

**(05 Marks)**

## Module-4

7 a. From the following table find the number of students who have obtained:

i) Less than 45 marks

ii) Between 40 and 45 marks.

| Marks | 30-40 | 40-50 | 50-60 | 60-70 | 70-80 |
|---|---|---|---|---|---|
| Number of students | 31 | 42 | 51 | 35 | 31 |

**(06 Marks)**

b. Construct the interpolating polgnomial for the data given below using Newton's general interpolation formula for divided differences and hence find y at x = 3.

| | | | | | | |
|---|---|---|---|---|---|---|
| x | 2 | 4 | 5 | 6 | 8 | 10 |
| y | 10 | 96 | 196 | 350 | 868 | 1746 |

**(05 Marks)**

c. Evaluate $\int_0^1 \frac{x}{1+x^2} dx$ by Weddle's rule. Taking seven ordinates. Hence find $\log_e 2$. **(05 Marks)**

**OR**

8 a. Use Lagrange's interpolation formula to find f(4) given below. **(06 Marks)**

| x | 0 | 2 | 3 | 6 |
|---|---|---|---|---|
| f(x) | -4 | 2 | 14 | 158 |

b. Use Simpson's 3/8$^{th}$ rule to evaluate $\int_1^4 e^{1/x}dx$. **(05 Marks)**

c. The area of a circle (A) corresponding to diameter (D) is given by

| D | 80 | 85 | 90 | 95 | 100 |
|---|---|---|---|---|---|
| A | 5026 | 5674 | 6362 | 7088 | 7854 |

Find the area corresponding to diameter 105 using an appropriate interpolation formula. **(05 Marks)**

## Module-5

9 a. Evaluate Green's theorem for $\phi_c(xy + y^2)\,dx + x^2dy$ where c is the closed curve of the region bounded by $y = x$ and $y = x^2$. **(06 Marks)**

b. Find the extremal of the functional $\int_a^b \left(x^2 + y^2 + 2y^2 + 2xy\right)dx$. **(05 Marks)**

c. Varity Stoke's theorem for $\vec{F} = (2x - y)\,\hat{i} - yz^2\hat{j} - y^2z\,\hat{k}$ where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ C is its boundary. **(05 Marks)**

**OR**

10 a. Derive Euler's equation in the standard form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y1}\right) = 0$. **(06 Marks)**

b. If $\vec{F} = 2xy\hat{i} + y^2z\hat{j} + xz\hat{k}$ and S is the rectangular parallelepiped bounded by $x = 0$, $y = 0$, $z = 0$, $x = 2$, $y = 1$, $z = 3$. Evaluate $\iint_S \vec{F}.\hat{n}\,ds$. **(05 Marks)**

c. Prove that the shortest distance between two points in a plane is along the straight line joining them or prove that the geodesics on a plane are straight lines. **(05 Marks)**

* * * * *

# CBCS SCHEME

USN | | | | | | | | | | |   **15ME32**

## Third Semester B.E. Degree Examination, Dec.2019/Jan.2020
## Material Science

Time: 3 hrs.  Max. Marks: 80

**Note:** ***Answer any FIVE full questions, choosing ONE full question from each module.***

### Module-1

1 a. Define the following : i) Unit cell ii) Lattice iii) Space lattice iv) Co-ordination number v) Automic packing factor. **(05 Marks)**
b. Differentiate between Edge and Screw dislocation. **(05 Marks)**
c. Explain factors affecting diffusion. **(06 Marks)**

**OR**

2 a. Explain surface defects with neat sketches. **(05 Marks)**
b. Explain cup and cone fracture, with neat sketch. **(05 Marks)**
c. Explain with neat stress – strain diagram, the mechanical properties of a material in plastic range. **(06 Marks)**

### Module-2

3 a. Explain solidification in pure metals. **(05 Marks)**
b. Define Solid Solutions. Explain the types of solid solutions. **(05 Marks)**
c. What is Hume Rothay's rule? Explain all the rules of Hume Rother's. **(06 Marks)**

**OR**

4 a. Lead ($P_b$) melts at 323$^0$C and tin ($S_n$) melts at 232$^0$C. Additions of $S_n$ to $P_b$ lowers the melting point Pb and addition of $P_b$ to $S_n$ also lowers the melting point of $S_n$ at 180$^0$C, Liquid of composition 61.9 % $S_n$, alpha ($\alpha$) phase of composition 19.2 % $S_n$ and beta ($\beta$) phase of composition 96.2 % $S_n$ are in thermal equilibrium. The solubilities of $P_b$ in $S_n$ and $S_n$ in $P_b$ at room temperature are negligible.
i) Draw the $P_b$ – $S_n$ diagram ii) Identify the reactions occurring at 180$^0$C
iii) Calculate the amount of phases in an alloy of composition 40% $S_n$ at 1790$^0$C. **(08 Marks)**
b. Draw the neat sketch of Iron – Carbon diagram. Indicate all phases and explain 3 invariant reactions. **(08 Marks)**

### Module-3

5 a. Explain T – T – T diagram for eutectoid steel. **(08 Marks)**
b. With a neat diagram, explain continuous cooling transformation diagram. **(08 Marks)**

**OR**

6 a. Define Heat treatment. List its objectives. **(05 Marks)**
b. Write the classification of heat treatment. **(05 Marks)**
c. Explain age hardening heat treatment for non – ferrous materials. **(06 Marks)**

### Module-4

7 a. What are Ceramics? List and explain processing of ceramics. **(07 Marks)**
b. Explain mechanical properties of ceramics. **(05 Marks)**
c. List advantages and applications of ceramics. **(04 Marks)**

**OR**

| | | | |
|---|---|---|---|
| 8 | a. | What are Plastics? List processing of thermoplastics and explain any one method. | **(07 Marks)** |
| | b. | Explain in brief the selection of engineering materials. | **(05 Marks)** |
| | c. | Explain NDT method for Residual life assessment. | **(04 Marks)** |

## Module-5

| | | | |
|---|---|---|---|
| 9 | a. | Define Composite Materials. List their classification based on matrix and reinforced constituents. | **(07 Marks)** |
| | b. | Differentiate between thermoset and thermoplastic materials. | **(05 Marks)** |
| | c. | Write a note on Metal Matrix Materials. | **(04 Marks)** |

**OR**

| | | | |
|---|---|---|---|
| 10 | a. | List and explain various fibers used in preparation of composite materials. | **(07 Marks)** |
| | b. | Explain Powder metallurgy technique of production of composite materials. | **(05 Marks)** |
| | c. | With neat sketch, explain Squeeze casting. | **(04 Marks)** |

* * * * *

CBCS SCHEME

USN | | | | | | | | | |

15ME33

# Third Semester B.E. Degree Examination, Dec.2019/Jan.2020
## Basic Thermodynamics

Time: 3 hrs. Max. Marks: 80

**Note: 1.** ***Answer FIVE full questions, choosing one full question from each module.***
***2. Use of thermodynamic data hand book is permitted.***





**Module-1**

1 a. Distinguish between the following:
(i) Microscopic and macroscopic point of view of thermodynamics.
(ii) Extensive and Intensive properties. **(05 Marks)**

b. Define the zeroth law of thermodynamics. A constant volume gas thermometer containing Helium gives a reading of gas pressure 'P' of 1000 mmHg and 1366 mmHg at ice point and steam point respectively. Assuming a linear relationship of the form, $t = \alpha + \beta P$, express the gas thermometer Celcius temperature 't' in terms of gas pressure 'P'. What is the temperature recorded by the thermometer when it registers a pressure of 1074 mmHg? **(06 Marks)**

c. Explain the thermodynamic definition of work with a suitable diagram. **(05 Marks)**

**OR**

2 a. Explain thermodynamic equilibrium concept. **(05 Marks)**

b. Deduce the expression for work in case of shaft work and electrical work. **(05 Marks)**

c. The combustion gases of an IC engine expand with in an enclosed piston and cylinder arrangement and follow the path $PV^{1.6} = C$. The pressure at the beginning of the power stroke is 5 MPa and volume 50 cm$^3$. At the end of the stroke the volume is 1500 cm$^3$. Calculate (i) The work developed during power stroke (ii) Average power developed by the gas if there are 20 power strokes per second. **(06 Marks)**

**Module-2**

3 a. Explain the Joule's experiment and describe how it leads to the foundation of first law of thermodynamics. **(05 Marks)**

b. A nozzle is used to convert enthalpy into kinetic energy. Air enters the nozzle at a pressure of 2700 KPa at a velocity of 30 m/s with an enthalpy of 923 KJ/kg and leaves with a pressure of 700 KPa and enthalpy of 660 KJ/kg. (i) If the heat loss is 0.96 KJ and mass flow rate is 0.2 kg/s, find the exit velocity (ii) Find the exit velocity for adiabatic conditions. **(07 Marks)**

c. State both Kelvin Planck and Clausius statements of thermodynamics. **(04 Marks)**

**OR**

4 a. Explain PMMI and PMMII with suitable diagrams. **(06 Marks)**

b. A reversible heat engine operates between two reservoirs at temperatures of 600°C and 40°C. The engine drives a reversible refrigerator which operates between the reservoir at temperature of 40°C and $-20^{\circ}C$. The heat transfer to the heat engine is 2000 KJ and the network output of the combined engine refrigerator plant is 360 KJ. Evaluate the heat transfer to the refrigerator and net heat transfer to the reservoir at 40°C. **(07 Marks)**

c. Comment on the limitations of the first law of thermodynamics. **(03 Marks)**

## Module-3

5 a. What are the factors, that makes a process irreversible? (05 Marks)

b. Derive Clausius inequality and comment on its outcome. (05 Marks)

c. A heat engine is shown in the Fig. Q5 (c) where 10000 KJ/hr of heat is supplied from source at 1400°C while the working fluid is at 540°C. 8000 KJ/hr of heat is rejected to a sink at temperature 5°C and working fluid is at 60°C. Calculate the following:

(i) Actual efficiency of the engine.

(ii) Fraction of the actual efficiency of the internally reversible efficiency.

(iii) Fraction of actual efficiency of the external reversible efficiency. (06 Marks)

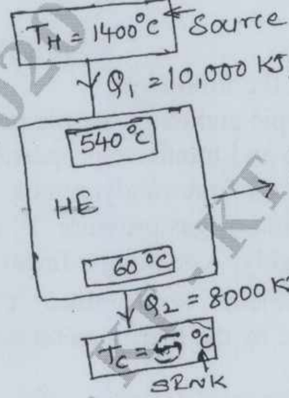



Fig. Q5 (c)

**OR**

6 a. Prove that entropy is a property. (05 Marks)

b. A closed system contains air at pressure 1 bar, temperature 290 K and volume 0.02 $m^3$. The system undergoes a thermodynamic cycle consisting of the following three processes: (i) Process 1 – 2 : constant volume heat addition till the pressure becomes 4 bar. (ii) Process 2 – 3 : Constant pressure cooling (iii) Process 3 – 1 : Isothermal heating to initial state. Represent the cycle in T-S and P-V plot. Evaluate the change of entropy for each case. Take $C_V = 718$ J/kg K, R = 287 J/kgK. (07 Marks)

c. Write a comment on thermodynamic temperature scale. (04 Marks)

## Module-4

7 a. Explain the concept of availability and unavailability. (04 Marks)

b. Explain the working of throttling calorimeter with a neat diagram. (05 Marks)

c. Find the maximum work/kg of air that can be obtained from a piston cylinder arrangement if the air expands from the initial state of $P_1 = 6$ bar, $t_1 = 170°C$ to a final pressure of $P_2 = 1.4$ bar, $t_2 = 60°C$. Neglecting changes in KE and PE and assuming $t_0 = 15°C$, calculate the availability in the initial and final states.
Compare the two results $(C_P)_{air} = 1.005$ KJ/kgK, R = 287 J/kg K. (07 Marks)

**OR**

8 a. Explain the PT diagram of a pure substance with all necessary points on it. (05 Marks)

b. Show that the change in availability is equal to the change in Gibbs function when the temperature and pressure of the system are constant. (05 Marks)

c. Steam from a boiler is delivered at 15 bar absolute and dryness fraction of 0.85 into a steam super heater where an additional heat is added at constant pressure. Steam temperature now increases to 573K. Determine amount of heat added and change in internal energy for unit mass of steam. (06 Marks)

## Module-5

9 a. Explain the Amagat's law of additive volume. **(05 Marks)**

b. State Vander Waal's equation of state and Beattie-Bridgeman equation. **(05 Marks)**

c. The air at DBT 28°C and 1 bar has a specific humidity of 0.016 kg per kg of dry air. Determine

(i) Partial pressure of water vapour.

(ii) Relative humidity.

(iii) Dew point temperature. **(06 Marks)**

**OR**

10 a. Explain the Dalton's law of partial pressures. **(04 Marks)**

b. Determine the pressure exerted by oxygen in a container of 2 $m^3$ capacity when it contains 5 kg at 27°C using (i) Ideal gas equation (ii) Vander Waals equation.

Take $a = 139.250 \dfrac{kNm^4}{(kgmole)^2}$; $b = 0.0314 \dfrac{m^3}{kgmole}$. **(07 Marks)**

c. Write short note on compressibility chart and its usefulness. **(05 Marks)**

* * * * *

**CBCS SCHEME**

USN | | | | | | | | | | — **15ME/MA34**

## Third Semester B.E. Degree Examination, Dec.2019/Jan.2020
# Mechanics of Materials

Time: 3 hrs. Max. Marks: 80

**Note:** ***Answer any FIVE full questions, choosing ONE full question from each module.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

## Module-1

1 a. Derive an expression for the extension of a uniformly tapering rectangular bar when it is subjected to an axial load P. **(08 Marks)**

b. Calculate the modulus of rigidity and bulk modulus of a cylindrical bar of diameter of 25mm and length 1.6m, if the longitudinal strain in a bar during a tension test is four times the lateral strain. Find the change in volume, when the bar is subjected to a hydrostatic pressure of $100N/mm^2$. Take $E = 1 \times 10^5 N/mm^2$. **(08 Marks)**

**OR**

2 a. A mild steel rod of 20mm diameter and 300mm long is enclosed centrally inside a hollow copper tube of external diameter 30mm and internal diameter of 25mm. The ends of the tube and rods are brazed together, and the composite bar is subjected to an axial pull of 40kN. If E for steel and copper is $200GN/m^2$ and $100\ GN/m^2$ respectively. Find the stresses developed in the rod and tube. Also find the extension of the rod. **(08 Marks)**

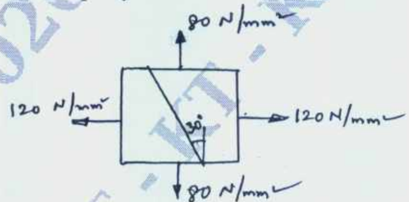
b. A steel bar is placed between two copper bars each having the same area and length as the steel bar at $15^0$C. At this stage, they are rigidly connected together at both the ends. When the temperature is raised to $315^0$C, the length of the bars increase by 1.5mm. Determine the original length and final stresses in the bars. Take $E_s = 2.1 \times 10^5\ N/mm^2$ ; $E_c = 1 \times 10^5\ N/mm^2$ ; $\alpha_s = 0.000012$ per $^0$C ; $\alpha_c = 0.0000175$ per $^0$C. **(08 Marks)**

## Module-2

3 a. Define Principal planes. Starting from the expression of normal and tangential stresses acting on inclined plane in an element subjected to 2D – stress state, derive the expressions for the magnitude and location of principal stresses. **(08 Marks)**

b. The direct stresses acting at a point in a strained material are as shown in fig. Q3(b). Find the normal, tangential and the resultant stresses on a plane $30^0$ to the plane of the major principal stress. Find also the obliquity of the resultant stresses. **(08 Marks)**

Fig.Q3(b)




**OR**

4 a. A thick walled cylindrical pressure vessel has inner and outer radii of 200mm and 250mm respectively. The material of the cylinder has an allowable stress of $75\ MN/m^2$. Determine the maximum internal pressure that can be applied and draw the sketch of radial pressure and circumferential stress distribution. **(08 Marks)**

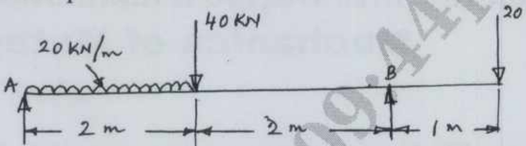
b. Derive expressions for circumferential Loop stress and longitudinal stress in thin cylinder. State the assumptions made in the derivation. **(08 Marks)**

## Module-3

5 a. Obtain the expressions for shear force and bending moment at a section of a cantilever beam carrying gradually varying load from zero at the free end to W per unit length at the fixed end. Draw the shear force and bending moment diagrams. **(06 Marks)**

b. Draw the shear force and bending moment diagrams for the overhanging beam shown in fig.Q5(b). Clearly indicate point of contra flexure. **(10 Marks)**

Fig.Q5(b)




**OR**

6 a. Derive the relation $\frac{M}{I} = \frac{\sigma_b}{Y} = \frac{E}{R}$ with usual notations and list the basic assumptions. **(10 Marks)**

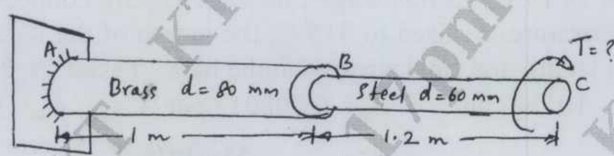
b. A simply supported beam of span 5m has a cross section 150mm × 250mm. If the permissible stress is $10N/mm^2$, find the maximum concentrated load P applied at 2m from one end, it can carry. **(06 Marks)**

## Module-4

7 a. Determine the diameter of a solid shaft which will transmit 300 KW at 250 rpm. The maximum shear stress should not exceed $30N/mm^2$ and twist should not be more than 1 in a shaft length of 2m. Take modulus of rigidity = $1 \times 10^5 N/mm^2$. **(08 Marks)**

b. The allowable shear stress in brass is $80N/mm^2$ and in steel $100N/mm^2$. Find the maximum torque that can be applied in the stepped shaft shown in fig. Q7(b). Find also the total rotation of free end with respect to the fixed end if $G_{brass} = 40\ kN/mm^2$ and $G_{steel} = 80kN/mm^2$. **(08 Marks)**

Fig.Q7(b)




**OR**

8 a. Find an expression for crippling load for a column with one end fixed and other end free. **(08 Marks)**

b. Determine the buckling load for a strut of T – section, the flange width being 100mm, overall depth 80mm and both flange and stem 10mm thick as shown in fig. Q8(b). The strut is 3m long and is hinged at both ends. $E = 200GN/m^2$. **(08 Marks)**

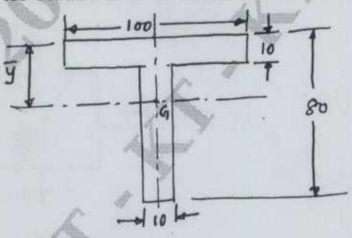



Fig.Q8(b)

## Module-5

9 a. Using Castiglione's first theorem, find the deflection at the free end of a cantilever beam carrying a concentrated load at the free end. Assume uniform flexural rigidity. **(06 Marks)**

b. Derive an expression for strain energy stored in a body due to torsion. **(10 Marks)**

**OR**

10 a. Write short notes on :
i) Maximum Principal stress theory ii) Maximum shear stress theory. **(10 Marks)**

b. A bolt is subjected to an axial pull of 12kN together with a transverse shear force of 6kN. Determine the diameter of the bolt by using Maximum principal stress theory. Take Elastic limit in tension = $300\ N/mm^2$, Factor of safety = 3. **(06 Marks)**

CBCS SCHEME

USN | | | | | | | | | | |

15MATDIP31

# Third Semester B.E. Degree Examination, Dec.2019/Jan.2020
## Additional Mathematics – I

Time: 3 hrs. Max. Marks: 80

***Note: Answer any FIVE full questions, choosing ONE full question from each module.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

### Module-1

1 a. Find modulus and amplitude of $1-\cos\theta+i\sin\theta$. **(05 Marks)**

b. Express $\dfrac{3+4i}{3-4i}$ in $a+ib$ form. **(05 Marks)**

c. Find the value of 'λ' so that the points A(−1, 4, −3), B(3, 2, −5), C(−3, 8, −5) and D(−3, λ, 1), may lie on one plane. **(06 Marks)**

**OR**

2 a. Find the angle between the vectors $\vec{a}=5\hat{i}-\hat{j}+\hat{k}$ and $\vec{b}=2\hat{i}-3\hat{j}+6\hat{k}$. **(05 Marks)**

b. Prove that $\left[\vec{a}\times\vec{b},\ \vec{b}\times\vec{c},\ \vec{c}\times\vec{a}\right]=\left[\vec{a}\,\vec{b}\,\vec{c}\right]^2$. **(05 Marks)**

c. Find the real part of $\dfrac{1}{1+\cos\theta+i\sin\theta}$. **(06 Marks)**

### Module-2

3 a. Obtain the $n^{th}$ derivative of $\sin(ax+b)$. **(05 Marks)**

b. Find the pedal equation of $r^n=a^n\cos n\theta$. **(05 Marks)**

c. If $u=\dfrac{yz}{x}$, $v=\dfrac{zx}{y}$, $w=\dfrac{xy}{z}$, show that $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}=4$. **(06 Marks)**

**OR**

4 a. If $u=\log\left(\dfrac{x^4+y^4}{x+y}\right)$ show that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=3$. **(05 Marks)**

b. If $u=f(x-y,\ y-z,\ z-x)$, show that $\dfrac{\partial u}{\partial x}+\dfrac{\partial u}{\partial y}+\dfrac{\partial u}{\partial z}=0$. **(05 Marks)**

c. If $y=a\cos(\log x)+b\sin(\log x)$, show that $x^2y_{n+2}+(2n+1)xy_{n+1}+(n^2+1)y_n=0$ **(06 Marks)**

### Module-3

5 a. Evaluate $\displaystyle\int_0^{\pi}x\sin^8x\,dx$. **(05 Marks)**

b. Evaluate $\displaystyle\int_0^1 x^2(1-x^2)^{3/2}dx$. **(05 Marks)**

c. Evaluate $\displaystyle\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}(x^2+y^2+z^2)\,dz\,dy\,dx$. **(06 Marks)**

**OR**

6 a. Evaluate $\int_0^1 \int_x^{\sqrt{x}} xy\,dy\,dx$. **(05 Marks)**

b. Evaluate $\int_0^1 \int_0^1 \int_0^1 (x+y+z)\,dx\,dy\,dz$. **(05 Marks)**

c. Evaluate $\int_0^\infty \frac{x^4}{(1+x^2)^4}\,dx$. **(06 Marks)**

## Module-4

7 a. If $\vec{r} = (t^2+1)\hat{i} + (4t-3)\hat{j} + (2t^2-6t)\hat{k}$, find the angle between the tangents at t = 1 and t = 2. **(05 Marks)**

b. If $\vec{r} = e^{-t}\hat{i} + 2\cos 3t\,\hat{j} + 2\sin 3t\,\hat{k}$, find the velocity and acceleration at any time t, and also their magnitudes at t = 0. **(05 Marks)**

c. Show that $\vec{F} = (y+z)\hat{i} + (z+x)\hat{j} + (x+y)\hat{k}$ is irrotational. Also find a scalar function 'φ' such that $\vec{F} = \nabla\phi$. **(06 Marks)**

**OR**

8 a. Find the unit normal vector to the surface $x^2y + 2xz = 4$ at (2, −2, 3). **(05 Marks)**

b. If $\vec{F} = xz^3\hat{i} - 2x^2yz\hat{j} + 2yz^4\hat{k}$ find $\nabla\cdot\vec{F}$ and $\nabla\times\vec{F}$ at (1, −1, 1). **(05 Marks)**

c. If $\frac{d\vec{a}}{dt} = \vec{w}\times\vec{a}$ and $\frac{d\vec{b}}{dt} = \vec{w}\times\vec{b}$, then show that $\frac{d}{dt}(\vec{a}\times\vec{b}) = \vec{w}\times(\vec{a}\times\vec{b})$ **(06 Marks)**

## Module-5

9 a. Solve $\sec^2 x\tan y\,dx + \sec^2 y\tan x\,dy = 0$. **(05 Marks)**

b. Solve $(y^3 - 3x^2y)dx + (3xy^2 - x^3)dy = 0$. **(05 Marks)**

c. Solve $\frac{dy}{dx} + \frac{y}{x} = xy^2$. **(06 Marks)**

**OR**

10 a. Solve $\frac{dy}{dx} + y\cot x = \cos x$. **(05 Marks)**

b. Solve $x^2y\,dx - (x^3+y^3)dy = 0$ **(05 Marks)**

c. Solve $y(x+y)dx + (x+2y-1)dy = 0$ **(06 Marks)**

* * * * *